RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 47, NUMBER 5 MAY 1993

Limited stochastic electron acceleration induced by an intense cyclotron wave in a plasma

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(Received 22 December 1992)

An investigation of the stochastic electron acceleration in a magnetized plasma due to an intense electron cyclotron wave is performed for the case of a stochastic regime where only a few resonances overlap and the stochastic region in phase space is bounded. The electron dynamics is described by a Hamiltonian function $H(\theta,I,t)$, and the stochastic properties of the system are investigated by means of the Poincaré-section method and the analytical estimates of the phase-correlation function. The behavior of the system is then analyzed through the Fokker-Planck-Kolmogorov (FPK) diffusion equation in action space. The theoretical FPK predictions are compared with the results of a numerical simulation of the particle motion. It is found that the system satisfies the diffusion equation at times that are short compared to the saturation time of the quasilinear diffusion. Appreciable deviations from the diffusive results are found at longer times, since the particle motion is influenced by the region of local stochasticity in phase space.

PACS number(s): 52.50.Gj, 52.35.Mw, 02.50.−r, 05.45.+b

The problem of the nonlinear interaction of a magnetized plasma with an intense beam of electromagnetic (em) waves has recently received increased interest, due to the development of powerful radio-frequency sources, such as gyrotrons and free-electron lasers, for plasma heating in controlled thermonuclear fusion devices [1]. The investigation of the nonlinearities induced in the particle dynamics by the em field is also of interest for the study of astrophysical plasmas and wave propagation in the ionosphere [2,3].

We are interested here in the stochastic particle motion and refer, for definiteness, to an electron cyclotron wave, with frequency ω, propagating in a plasma perpendicularly to a static magnetic field $\mathbf{B}_0=B_0\mathbf{e}_z$. The perturbation parameter relevant to the problem is $\epsilon=eE/mc\omega<1$, with E the electric-field amplitude of the wave. For vanishingly small ϵ the wave interacts resonantly with the electrons, whose energy $mc^2\gamma$ satisfies the condition $\omega=n\Omega/\gamma$, where $\Omega=eB_0/mc$ is the nonrelativistic electron gyrofrequency and n is a positive integer. If ϵ is sufficiently high a stochastic regime can set in, because of the resonance overlapping, which is driven by the strong em perturbation. In the following, we consider explicitly the case of the extraordinary mode at the second cyclotron harmonic, since the conditions for stochasticity are less severe in comparison with the case of the ordinary mode [1,4].

The analysis of this dynamical system in the globally stochastic regime, where a large number of resonances overlap, has been made in detail in Ref. [4], where it has been shown that the energy variation process is diffusive, and the theoretical predictions, obtained on the basis of a Fokker-Planck-Kolmogorov (FPK) equation with a local quasilinear diffusion coefficient, are in good agreement with the results from direct numerical integration of the equations of motion for a set of particles.

Here we consider a different physical situation, not yet investigated in the literature, where only a few resonances overlap and at high energy a transition to regular motion occurs. For the mode under consideration at a given ϵ this regime is found when the plasma density is increased, due to the corresponding modification of the wave polarization. The aim of the present paper is to investigate the electron-energy-gain process in this case, and in particular the validity of a FPK approach. The method followed is quite general, and the main results found here can be applied to other cases where the region of stochastic motion in phase space is bounded.

In the system under consideration, the electron motion is described by means of a relativistic Hamiltonian $H(I,\theta,t)$ where θ,I are angle-action variables. To first order in ϵ it reads [4]

$$H=\gamma=H_0(I)+H_1(\theta,I,t)\ , \tag{1}$$

where

$$H_0=\Gamma=(1+2I+P_z^2)^{1/2}\ ,$$

$$H_1=-\epsilon\sum_n H_{1n}(I,P_z)\cos(n\theta-\nu t)\ ,$$

where $\nu=\omega/\Omega$, $H_{1n}=\Theta_n(P_z,I)/\Gamma$, and the momentum P_z conjugate to z is a constant of motion. The polarization term reads $\Theta_n=\sqrt{2I}\,[\eta_- J_{n-1}(b)+\eta_+ J_{n+1}(b)]+P_z\eta_z J_n(b)$, where η_-, η_+, and η_z, are the right, left, and parallel components of the polarization vector, J_n is the Bessel function of order n and argument $b=\nu N\sqrt{2I}$, and $N=kc/\omega$ is the refractive index. For the unperturbed system, θ represents the phase of the electron gyromotion, and $I=p_\perp^2/2$. Dimensionless quantities are used: time, length, momentum, energy, and vector potential are normalized over Ω^{-1}, $c\Omega^{-1}$, mc, mc^2, and mc^2/e, respectively.

A globally stochastic regime takes place in the plasma for sufficiently large ϵ values: $\epsilon>\epsilon_c$, ϵ_c being the (action dependent) critical value for the transition to stochastic motion, which can be estimated on the basis of the Chirikov criterion of overlapping resonances [5]. For our system $\epsilon_c\simeq[16\nu^2\Theta_n(\bar{I}_n)]^{-1}$, where $\bar{I}_n=1/2(n^2/\nu^2-1-P_z^2)$

is the resonant action value [4]. In the limit of negligible density ϵ_c is proportional to $I^{-1/6}$. At finite density, ϵ_c exhibits a minimum as a function of I. The action value corresponding to it decreases as the plasma density increases. Then, for a given ϵ, the condition $\epsilon_c(I)=\epsilon$ defines an action value I_M, which gives an estimate of the upper action boundary in phase space.

A qualitative description of the behavior of the system can be given by means of the well-known Poincaré surface of section method, which in our case is obtained plotting the values of θ, I at the times $t_k=2\pi k/\nu$, with $k=0,1,\ldots$, i.e., at times that are multiples of the wave period. A typical plot is shown in Fig. 1 that is relevant to a large-density case. We note that the Poincaré section of the system under consideration cannot be obtained by a simple mapping, due to a nontrivial dependence of the Hamiltonian H_1 on the action. The surface of section can be roughly divided into three distinct zones: at low action values, a region of globally stochastic dynamics is found, followed by a region of local stochasticity, where the coexistence of regular structures and chaotic orbits is apparent, and finally by a region of regular (nonstochastic) dynamics. On the long time scale, the behavior of the system is expected to be strongly affected by the existence of the intermediate region of vanishing diffusion and locally stochastic dynamics. Particles are partly "reflected," and partly suffer a slow diffusion through this transition region. Moreover, the presence of the upper region of regular dynamics gives a limit in the energy the electron population can gain. The boundary between the region of regular motion and the region of local stochasticity is characterized by the presence of invariant curves, which exhibit large oscillations in action. The structure of these curves can be easily explained as an interference process between two adjacent resonances of high order. It is found that the upper limit I_M given by the Chirikov criterion is in qualitative agreement with the results of the Poincaré plot.

For the system described by the Hamiltonian (1), a diffusion equation for the distribution function averaged over the phases $f(I,t)$ can be written, following the FPK approach [6]

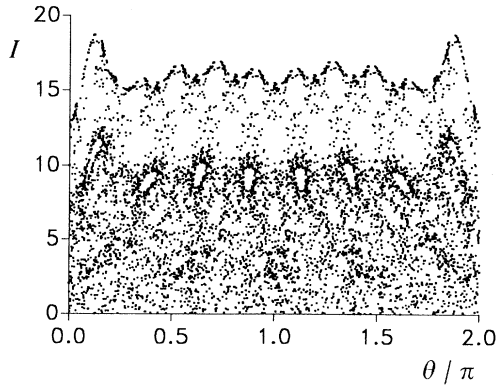

FIG. 1. Poincaré plot for the dynamical system described by the Hamiltonian (1). The chosen values of the parameters are $\nu=\omega/\Omega=1.8$, $\omega_p/\omega=0.6$, and $\epsilon=0.4$. Six initial conditions at $I_0=10$, $P_z=0$, and uniform phase θ_0 have been considered.

$$\frac{\partial f}{\partial t}=\frac{1}{2}\frac{\partial}{\partial I}D(I)\frac{\partial f}{\partial I}\ . \tag{2}$$

To lowest order in ϵ, this equation describes a diffusion process in the variable $p_\perp^2/2$, at constant P_z.

Taking explicitly into account the effects of the phase correlations, D can be written as

$$D(I)=\pi\epsilon^2\sum_n n^2H_{1n}^2\Delta_n(\alpha_n)\ , \tag{3}$$

where

$$\alpha_n=n/\Gamma-\nu\ ,$$

$$\Delta_n=(1/\pi)\mathrm{Re}\int d\tau\exp(i\alpha_n\tau)R_n(\tau)\ ,$$

$$R_n(\tau)=\exp(-in\tau/\Gamma)\int_0^{2\pi}\frac{d\theta_0}{2\pi}\exp\{in[\theta(\tau)-\theta_0]\} \tag{4}$$

is the phase correlator, and the average in Eq. (4) is performed over the initial phases θ_0.

The correlation function R_n has been estimated analytically, linearizing the equation of motion around a given initial condition θ_0, I_0:

$$R_n=\prod_{m\ (\neq 0)}J_0[\epsilon n(A_m^2+B_m^2)^{1/2}]\ , \tag{5}$$

where

$$\begin{aligned}A_m&=(\alpha_m'/\alpha_m)H_{1m}t\\&\quad-[(\alpha_m'/\alpha_m)H_{1m}-H_{1m}']\sin(\alpha_m t)/\alpha_m\ ,\\B_m&=(\alpha_m'/\alpha_m)H_{1m}-H_{1m}'][1-\cos(\alpha_m t)]/\alpha_m\ ,\end{aligned} \tag{6}$$

and the prime denotes the derivative with respect to the action. The expression (6) differs from that given in Ref. [4] since now the explicit dependence of H_{1n} on the action has been taken into account. The behavior of $R_n(t)$, as obtained by direct numerical integration of the equations of motion and by Eq. (5), is shown in Fig. 2 for two different values of the initial action. The agreement be-

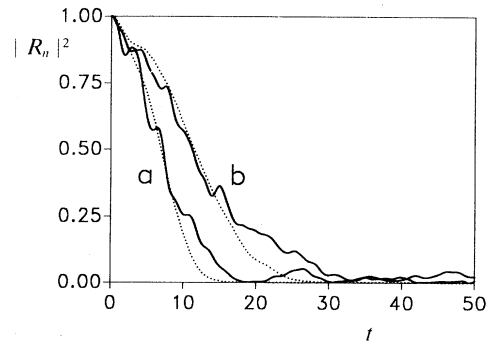

FIG. 2. Behavior of the modulus squared of the correlation function $|R_n|^2$ vs time. The dotted curve represents the analytical expression derived from Eq. (5). The solid curve is obtained from the numerical solution of the equations of motion for a set of particles. The number of particles is $N_P=500$, and a uniform random phase distribution has been chosen. The values of the parameters are the same as in Fig. 1. Curves a and b correspond to an initial action value $I_0=5$ and 10, respectively.

tween the numerical and the analytical results is fairly good at short times (up to approximately the first zero), while at longer times the analytical expression slightly differs from the numerical curve, since the linearization procedure is not valid. The characteristic correlation time t_c can be estimated as the first root of the function $R_n(t)$. It is found [4] that it scales approximately as $t_c \simeq 8\gamma(\epsilon_c/\epsilon)^{1/2}$. When the effect of a finite correlation time is taken into account, the diffusion coefficient can be explicitly computed by inserting Eq. (5) in Eq. (3).

For large correlation times $\Delta_n \rightarrow \delta(\alpha_n)$ and Eq. (3) reduces to

$$D(I)=\pi\epsilon^2 \sum_n n^2 H_{1n}^2 \delta\left[\frac{n}{\Gamma}-\nu\right], \tag{7}$$

which represents the well-known quasilinear result. An effective diffusion coefficient, which is valid around each resonant value of the action $\bar{I}_n$, can be found by averaging $D(I)$, as given by Eq. (7), over the region between two adjacent resonances, thus obtaining [4]

$$D_n \equiv \langle D(\bar{I}_n)\rangle = \pi\epsilon^2 (n^3/\nu) H_{1n}^2(\bar{I}_n) . \tag{8}$$

The smoothing of this function defines a local quasilinear diffusion coefficient $D_{\mathrm{QL}}(I)$.

In Fig. 3, the expression (3) for the diffusion coefficient is shown as a function of the action, together with D_{QL}. Since the correlation time is quite large, $D(I)$ shows strong oscillations peaked around each harmonic, with an average value given approximately by D_{QL}. The local quasilinear diffusion coefficient D_{QL} has a single maximum and then decreases up to the action value I_M. Beyond this value, it is assumed to be zero, since the dynamical system is no longer governed by global stochasticity and exhibits the characteristic behavior of the transition to chaos.

In order to test the predictions of the theoretical model, numerical simulations of the motion have been performed for a set of $N_P=1500$ particles with uniformly distributed initial phases. The chosen parameters are $\nu=\omega/\Omega=1.8$, $\omega_p/\omega=0.6$, $P_z=0$, and $I_0=1$. The evolution of the system has been followed for a very long time (up to $t=4000$) compared to the correlation time. The

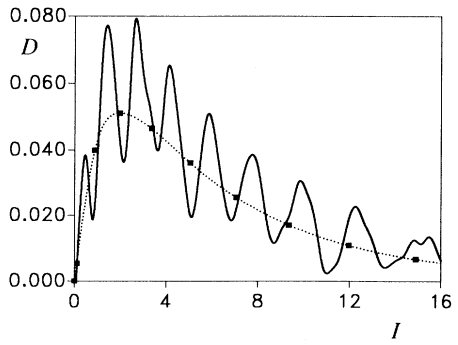

FIG. 3. Behavior of the diffusion coefficient D as a function of the action I. The solid curve represents $D(I)$ given by Eq. (3), the dotted curve represents D_{QL}, and the squares represent the values D_n. The same parameters as in Fig. 1 are considered.

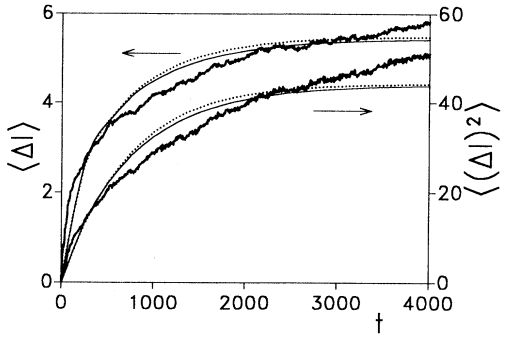

FIG. 4. Average action deviations $\langle \Delta I\rangle$ and $\langle(\Delta I)^2\rangle$ vs time. The solid line is the result of the numerical simulation; the thin solid line and the dotted line are obtained solving the FPK equation with the coefficient given by Eq. (3) and with D_{QL}, respectively. The initial condition $I_0=1$ and a uniform random phase distribution have been chosen. The upper action boundary $I_M=13$ has been chosen. The number of electrons is $N_P=1500$. The other parameters are the same as in Fig. 1.

numerical energy distribution function is obtained by building the histogram with energy step $\Delta\gamma=1/\nu$, which corresponds to the energy interval between two adjacent resonances. The average variations of the action are computed as $\langle \Delta I\rangle=\sum_{i=1}^{N_P}[I_i(t)-I_i(0)]/N_P$ and $\langle(\Delta I)^2\rangle=\sum_{i=1}^{N_P}[I_i(t)-I_i(0)]^2/N_P$. The results of the numerical simulations are compared with those obtained by the solution of the diffusion equation (2), both with the local quasilinear diffusion coefficient and the theoretical estimate (3). Equation (2) has been solved with the conditions of zero flux $D\partial f/\partial I$ at the boundaries $I=0$ and $I=I_M$. The average action deviations versus time and the energy distribution function at four different times are plotted in Figs. 4 and 5, respectively.

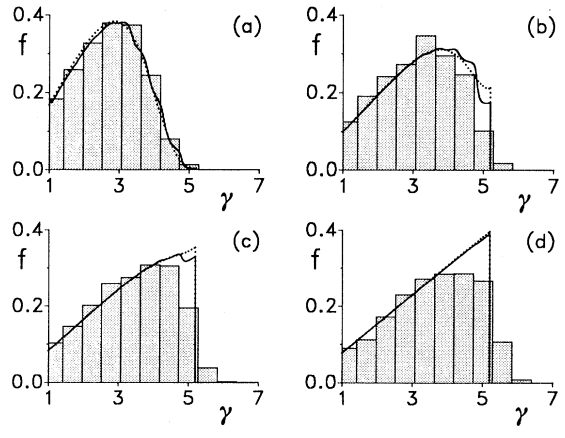

FIG. 5. Distribution function f as a function of the energy γ, for four different values of the time. The histogram represents the distribution function obtained by the simulation of the motion. The solid curve and the dotted curve are the solution of the FPK equation with coefficient given by Eq. (3) and with D_{QL}, respectively. The parameters are the same as in Fig. 4. Cases (a), (b), (c), and (d) refer to $t=250$, 1000, 2000, and 4000, respectively.

A very good agreement is found between the results of the numerical simulation of motion and the FPK results at relatively short times (for the chosen parameters up to $t \simeq 500$) but still much longer than the phase correlation times. The distribution function obtained by the numerical simulation shows a spread in energy that is in agreement with a diffusive behavior (recall that the initial condition is peaked at $\gamma \simeq 1.8$). At longer times the behavior of the system is no longer diffusive in the whole phase space, since the particle motion is influenced by the region of local stochasticity, which is found at large action value (see Fig. 1). Therefore, the average action deviations are lower than the predicted FPK values. At even longer times ($T \gtrsim 2500$) the FPK approach foresees a saturation of the average values of the action and of the distribution function, which are independent of the specific value and shape of the diffusion coefficient. The saturation process is due to the presence of the upper boundary I_M: $f(I)$ approaches the constant value I_M^{-1} [$f(\gamma)$ scales as γ]. On the contrary, no saturation of f on the whole stochastic region is observed up to the very long time scale of the simulation. The actual process turns out to be only partially diffusive. Moreover, a particle leakage is observed through the boundary $I = I_M$ determined by the Chirikov criterion.

Comparing the results obtained solving the diffusion equation (2) with the local quasilinear diffusion coefficient D_{QL} and with the coefficient (3), which takes into account the correlation effects, negligible differences are found in the average action deviations. The distribution function computed with the diffusion coefficient (3) exhibits small oscillations around the quasilinear function, which are more pronounced at large γ, where t_c is longer, and at short times, i.e., before the occurrence of the saturation.

In conclusion, we have analyzed the electron acceleration by an electromagnetic wave in a case where the stochastic region in phase space is bounded. We have identified two well-defined phases of the process: a diffusive phase, characterized by a globally stochastic dynamics in phase space and well described by a quasilinear diffusion coefficient with an upper limit determined by the Chirikov criterion; a subsequent phase, where a local regular motion manifests itself through longer correlation times and through the presence of islands and boundaries in phase space, so that the diffusive character of the process is lost.

The Istituto di Fisica del Plasma is "EURATOM-ENEA-CNR Association."

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